Q-compensation PSDM in the frequency domain

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Summary

In anelastic median different frequency components of seismic waves propagate in different velocities and encounter energy attenuation in varying degrees. Without properly taking into account of these factors, conventional images of PSDM suffer in phase distortion and reduced resolution for lower horizons. We developed a prestack depth migration (PSDM) method in the frequency domain using precisely the frequency dependent velocity model. For frequency dependent amplitude compensation, gain controlled approaches are employed to stabilize the process and to suppress noises in frequency and depth dependent manner. Synthetic and real date examples demonstrate the correct phase and improved resolution in images produced by this Q compensation PSDM.

Introduction

It is well known that the earth strata are far from perfectly elastic but demonstrate the properties of viscoelasticity, in which seismic waves suffer energy loss during propagation. Seismic attenuation is commonly characterized by the quality factor $Q$ defined in terms of mean energy stored during a deformation cycle or wavelength divided by the energy lost during the cycle. The linear model of wave attenuation with frequency independent $Q$ is widely used in exploration seismology (Kjartansson, 1979). Based on this theory, velocity dispersion exists in anelastic materials as a phase velocity can be expressed as a function of frequency in contrast to the frequency independent phase velocity for elastic media. As for energy loss, the attenuation coefficient is approximately proportional to the frequency. In general, higher frequency components in seismic waves tend to travel faster, and their amplitude decays more quickly. Seismic processing without properly taking into account of wave dispersion and dissipation results in images with distorted phases and reduced resolutions, especially for deeper horizons under low quality factor (low $Q$) strata, leading to problems in interpretation such as AVO analysis and matching seismic horizons to well data.

Early developed processing methods mitigating anelastic effects are applied to seismic traces before migration. Based on one-dimensional wave modeling, these attempts include phase-only inverse $Q$-filtering (e. g. Bickel and Natarajan, 1985; Bano, 1996) as well as full inverse $Q$-filtering that performs amplitude compensation and phase correction simultaneously (Wang 2006). Since these methods do not trace the actual wave pathand do not account for the fact that velocity dispersion and energy loss occur along with wave propagation, they are not suitable for correcting the dissipation effects in data from complex geology areas. An ideal way is to apply $Q$-compensation in prestack depth migration methods based on wavefield simulations (Zhang et al., 2010.; Zhu et al., 2014). Because of the frequency dependency of both velocity dispersion and energy attenuation in anelastic media, it is natural and straightforward to implement such migration in the frequency domain. Earlier work in this area includes an inverse $Q$ migration by Dai and West (1994) and a PSDM by wavefield extrapolation (Zhang and Wapenaar, 2002). However, some of these proposals did not thoroughly resolve the numerical instability problems caused by amplification operations in the wavefield extrapolation that is required by the compensation...
for energy absorption. Wang (2008) proposed an inverse-$Q$ filtered migration method for post-stack seismic data. He derived a stabilized attenuation coefficient by constraining the gain of amplitude compensation to a pre-defined level for higher frequencies and greater depth. The stabilized attenuation coefficient is then used to build a frequency-dependent, complex-valued slowness model, which in turn is utilized in an implicit finite difference scheme to perform the migration.

In this paper, we describe our implementation of $Q$ compensation PSDM in the frequency domain. Frequency dependent velocity model, as a function of spatially varying $Q$ model, is used in wavefield extrapolation. For full $Q$-compensation migration, in the sight of the concept of stabilized attenuation coefficient (Wang, 2006), we designed a gain control as a function of frequency and depth to stabilize the extrapolation procedure and to suppress amplified high-frequency noises. Synthetic and real data examples demonstrate the enhancement of the resulting images in terms of resolution and phase fidelity.

**Method**

We start from a one-way wave equation

$$\frac{\partial u(k_x,k_y,z,\omega)}{\partial z} = i k_z u(k_x,k_y,z,\omega),$$

where $k_x$, $k_y$, $k_z$ are the wavenumber scomponents in the $x$, $y$, $z$ directions, respectively, $u(k_x,k_y,z,\omega)$ is a plane wave of angular frequency $\omega$ at depth $z$. A solution of wave equation (1) is

$$u(k_x,k_y,z + \Delta z,\omega) = \exp (i k_z \Delta z) u(k_x,k_y,z,\omega).$$

Denoting $s$ as the slowness of wave propagation, $k = s\omega$ is the wavenumber, while $k_z$ is defined as

$$k_z = s\omega \sqrt{1 - (k_x^2 + k_y^2)/k^2}. \quad (3)$$

Equation (2) extrapolate the wavefield from depth $z$, over a depth step $\Delta z$, to depth $z+\Delta z$. Based on the linear model for attenuation of seismic waves with frequency independent $Q$ factor (Kjartansson, 1979), for a visco-acoustic media, the real-number slowness $s$ is replaced by a frequency-dependent complex-number slowness $S(\omega)$, defined as

$$S(\omega) = s(\omega) - i \alpha(\omega), \quad (4)$$

where

$$s(\omega) = s_o \left( \frac{\omega}{\omega_o} \right)^\gamma, \quad (5)$$

$$\alpha(\omega) = \frac{s_o}{2Q} \left( \frac{\omega}{\omega_o} \right)^\gamma. \quad (6)$$

$s_o$ is the propagation slowness at a reference frequency $\omega_o$, while $s(\omega)$ is the dispersive slowness and $\omega\alpha(\omega)$ is known as the attenuation coefficient and

$$\gamma = \frac{1}{\pi} \tan^{-1} \left( \frac{1}{\pi Q} \right) \approx \frac{1}{\pi Q}. \quad (7)$$

For $Q^2 \gg 1$, the vertical wavenumber in equation (3) becomes

$$k_z = [s(\omega) - i \alpha(\omega)]\omega \sqrt{1 - (k_x^2 + k_y^2)/k^2}. \quad (8)$$

The real part of $k_z$ is related to the dispersive propagation of waves, while the imaginary part is associated with the frequency-dependent wave energy absorption in the attenuation media.

We implement the wavefield extrapolation (2) for visco-acoustic media using a split-step plus
interpolation scheme (Dai et al., 2002, Gazdag and Sguazzero, 1984; Stoffa et al., 1990), where the amplitude-scaled phase-shift is performed in the dual domains of wavenumber and space. For a frequency sample, at each depth level, a number of reference slowness values $s_i$ and $Q_j$ (hence $\alpha_j$) are selected. First, we use equation (2) and a modified vertical wavenumber $k_z'$ as given in equation (9)

$$k_z' = [s_i(\omega) - i\alpha_j(\omega)]\omega \left(\sqrt{1 - (k_x^2 + k_y^2)/k^2} - 1\right)$$

(9)

To obtain a number of reference wavefields. This operation keeps the outcomes at the origin of the horizontal wavenumber plane unchanged. After being Fourier transformed to the spatial domain, the results are used for interpolation based on the local slowness $s_0(x, y, z)$ and $Q$-factor $Q(x, y, z)$ to produce an intermediate wavefield. It is then followed by an amplitude-scaled phase-shift in the spatial domain, using $k_{z0}$, the vertical wavenumber at the origin of horizontal wavenumber plane:

$$k_{z0} = [s(x, y, z, \omega) - i\alpha(x, y, z, \omega)]\omega$$

(10)

The extrapolation operator as a whole is an amplifier because its scale factor is greater than 1 for a finite positive $Q$-value. It gives rise to an instability problem because the high-frequency noises including the computer round-off errors can grow exponentially with the increasing depth.

To stabilize the extrapolation procedure, we follow Wang (2008) to derive a stabilized attenuation coefficient tuned by a parameter that relates to a maximum gain in amplitude. Base on a solution of the one-dimension wave equation in visco-acoustic media, the amplitude attenuation factor, can be written as

$$W(n, \omega) = \exp[-\omega\Delta z \sum_{l=1}^{n} \alpha(z_l, \omega)]$$

(11)

A stabilized amplitude-compensation scaler is defined as

$$W^{-1}(n, \omega) = \frac{W(n, \omega) + \sigma^2}{W^2(n, \omega) + \sigma^2}$$

(12)

The stabilization parameter $\sigma$ is related to a user-specified gain limit by an empirical relationship (Wang, 2006). A stabilized attenuation coefficient $\omega \alpha_{st}(z_l, \omega)$ is then defined by a recursion relation as

$$\exp[\omega\Delta z \sum_{l=1}^{n} \alpha_{st}(z_l, \omega)] = W^{-1}(n, \omega)$$

(13)

with $\alpha_{st}(z_1, \omega) = \alpha(z_1, \omega)$ A three-dimensional model is obtained by putting all the 1D attenuation coefficient function at each horizontal position together. It is the stabilized attenuation coefficient $\omega \alpha_{st}(z_l, \omega)$ that is used in the wavefield extrapolation to stabilize the procedure.

Examples

We created a 3D shot-gather of synthetic seismic data using a finite-difference method based on a three-layer model with faults. The median is visco-acoustic with a constant $Q = 30$. In Figure 1, we show the images of 3D PSDM results of the one-shot data. Figure 2 shows the zoom-in image of the central portion of the first horizon in Figure 1. Figure 1a and 2a are the same output of a conventional acoustic one-way wave equation PSDM. The wavelets in these images are asymmetrical although the source wavelet used in the finite difference modeling is symmetrical. Figure 1b and 2b show the result of our $Q$-compensation PSDM with only the velocity dispersion correction applied. The wavelets in these images are now symmetrical.
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Figure 1. PSDM image of 1 shot gather, (a) by conventional acoustic one-way wave-equation migration; (b) by the Q-compensation migration with only the dispersion correction; (c) by the full Q-compensation migration.

Figure 2. The zoom-in images of the central portion of the first horizon in Figure 1, (a) by conventional acoustic one-way wave-equation migration; (b) by the Q-compensation migration with only the dispersion correction; (c) by the full Q-compensation migration.

Figure 3. PSDM image of a real 3D data set, (a) by conventional acoustic one-way wave-equation migration; (b) by the full Q-compensation migration.
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![Figure 4. Amplitude spectra of the image in Figure 3a (red) and of the image in Figure 3b (blue).](image1)

![Figure 5. A Q-factor model that is used to produce image in Figure 6b.](image2)

![Figure 6. PSDM image of a real 3D data set, (a) by conventional acoustic one-way wave-equation migration; (b) by the full Q-compensation migration.](image3)

and also positioned at the rightdepth. Figure 1c and 2c are images from the full Q-compensation PSDM. The wavelets of this image are not only symmetrical, at the correct depth but also with much shorter time-duration hence uplifted resolution.

A real data example is shown in Figure 3. The two results in Figure 3a and Figure 3b are generated by using the same computer program. Figure 3a is the conventional PSDM image with the Q-compensation option turned off, and Figure 3b is the output of the 3D full Q-compensation PSDM with a maximum gain of amplitude controlled at 25 dB. Figure 3b shows much more detailed interpretable seismic characteristics with slightly modified horizon depths and phases compared to the image in Figure 3a. The amplitude spectra of the images in Figure 3a and Figure 3b are shown in Figure 4 on the same un-normalized scale for comparison. The spectrum comparison clearly demonstrates the broader bandwidth of the image from the full Q-compensation PSDM.

In another real data example, normal faults form a
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graben basin where deposited are unconsolidated sediments in the shallow section. Within the basin is a low \(Q\)-factor zone where seismic waves encounter severe attenuation (Figure 5). In Figure 6a the image of conventional PSDM shows weak seismic events with reduced resolution in and beneath the low \(Q\)-factor zone. In contrast, in Figure 6b, the image resulting from the full \(Q\)-compensation PSDM presents enhanced resolution and much stronger seismic energy on the horizons within and under the severe attenuation graben.

**Conclusions**

We presented a newly developed \(Q\)-compensation PSDM method in the frequency domain. The method corrects the phase distortion caused by the velocity dispersion in anelastic media by wavefield extrapolation using the exact frequency-dependent velocity model. For a full \(Q\)-compensation PSDM including amplitude effects, stabilization attenuation coefficient models are built to stabilize the wave extrapolation. Synthetic and real data examples demonstrate that the \(Q\)-compensation PSDM method not only produces images with horizons positioned at the right depths and with wavelets of corrected phases but also broadens the spectra of the images and uplifts the seismic resolution.

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**References**


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